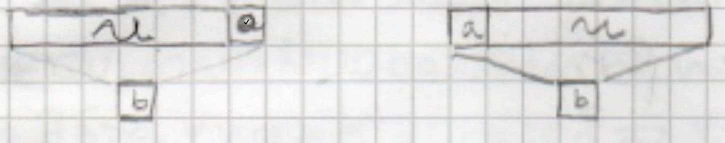
<u>DEF</u> PERMUTIVITY (PERMUTATIVITY)

$f: A^{2r+1} \to A$ is RIGHTMOST-PERMUTIVE (resp. LEFTMOST-PERMUTIVE)

iff $\forall u \in A^{2r}$ $\forall b \in A$ $\exists! a \in A$ t.c. $f(ua) = b$ $\big($resp. $f(au) = b\big)$



R-permutivity L-permutivity

<u>REMARK</u> Establishing if a rule is R-permutive or L-permutive is decidable

EXAMPLES

$m_f = 170$

$\{0,1\}^3$	f
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

$m_f = 170$

$\{0,1\}^3$	f
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

$m_f = 90$

$\{0,1\}^3$	f
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	1
1 0 1	0
1 1 0	1
1 1 1	0

$m_f = 106$

$\{0,1\}^3$	f
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	1
1 1 1	0

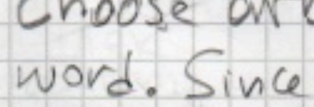
with $m_f = 170$ $F = \sigma$
f is R-perm.
f is not L-perm

with $m_f = 90$
f is R-perm
f is L-perm

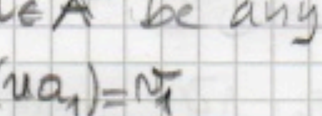
with $m_f = 106$
f is R-perm
f is not L-perm

<u>PROPOSITION</u> If f is either R-perm or L-perm $\Rightarrow$ F is surjective

<u>proof</u> We are going to show that $\forall v \in A^+$ $|F^{-1}(v)| = |A|^{2r}$
Choose arbitrarily $v \in A^m$ and let $u \in A^{2r}$ be any word. Since f is R-perm, $\exists! a_1 \in A$ s.t. $f(ua_1) = v_1$



Then $\exists! a_2 \in A$ s.t. $f(\boxed{u_2 \; u_n a_1} \, a_2) = v_2$



... $\exists! a_m \in A$ s.t. $f(\boxed{u_n a_1 \; a_{n-1}} \, a_n) = v_m$. Therefore, $\forall u \in A^{2r}$
$\exists! \alpha = a_1 \cdots a_m$ s.t. $f(u\alpha) = v$. Hence, v has 1 pre-image $\forall u \in A^{2r}$