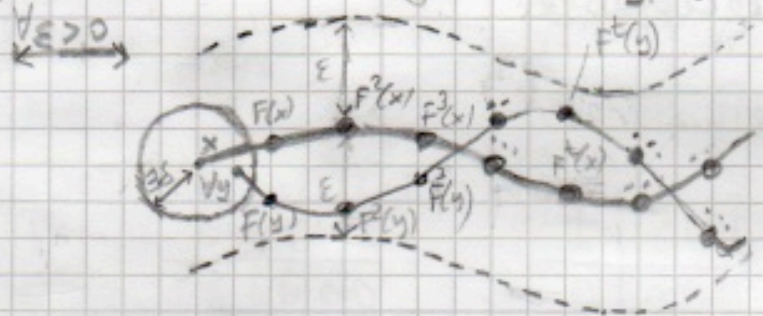
STABILITY - INSTABILITY FOR ANY DTDS

The following notions are valid for any (D)ISCRETE (T)IME) (D)YNAMICAL (S)YSTEM, i.e., for any pair (X,F) where (X,d) is a metric space and $F: X \to X$ is a continuous transformation.

<u>DEF</u> STABILITY (EQUICONTINUITY) (LYAPUNOV STABLE) POINT

$x \in X$ is a stability (equicontinuity) (Lyapunov stable) point

iff $\forall \varepsilon > 0\ \exists \delta > 0\ \forall y \in X\ d(y,x) < \delta \Rightarrow \forall t \in \mathbb{N}\ d(F^t(y), F^t(x)) < \varepsilon$



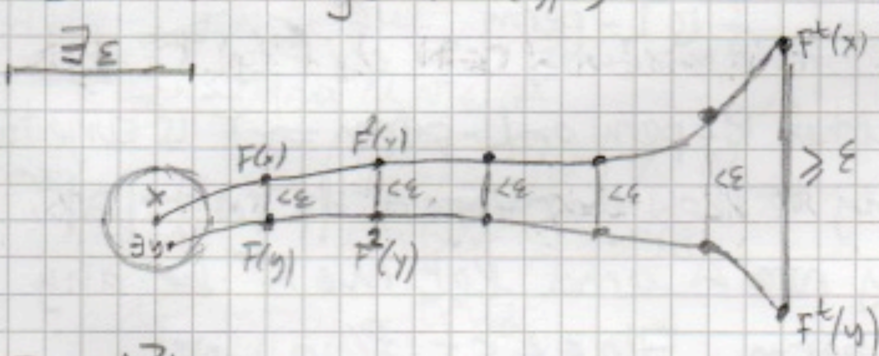
When $X = A^{\mathbb{Z}}$ and d is the Tychonoff distance one can equivalently write

$\forall n \in \mathbb{N}\ \exists m \in \mathbb{N}\ \forall y \in A^{\mathbb{Z}}\ y_{[-m,m]} = x_{[-m,m]} \Rightarrow \forall t \in \mathbb{N}\ F^t(y)_{[-n,n]} = F^t(x)_{[-n,n]}$

<u>DEF</u> UNSTABLE POINT (INSTABILITY POINT)

$x \in X$ is an unstable point if x is NOT a stability point

$\exists \varepsilon > 0\ \forall \delta > 0\ \exists y \in X\ d(y,x) < \delta \wedge \exists t \in \mathbb{N}\ d(F^t(y), F^t(x)) \geq \varepsilon$



In $A^{\mathbb{Z}}$

$\exists n \in \mathbb{N}\ \forall m \in \mathbb{N}\ \exists y \in A^{\mathbb{Z}}\ y_{[-m,m]} = x_{[-m,m]} \wedge \exists t \in \mathbb{N}\ F^t(y)_{[-n,n]} \neq F^t(x)_{[-n,n]}$

DEF STABLE SYSTEM (GLOBAL STABILITY)

(X,F) is STABLE if $\forall x \in X$, x is a STABILITY POINT

$\forall x \in X \underbrace{\forall \varepsilon > 0 \exists \delta > 0 \forall y \in X\, d(y,x) < \delta \Rightarrow \forall t \in \mathbb{N}\, d(F^t(y), F^t(x)) < \varepsilon}_{x \text{ IS STABLE}}$

DEF EQUICONTINUOUS SYSTEM (EQUICONTINUITY)

$\forall \varepsilon > 0 \exists \delta > 0 \forall y, x \in X\, d(y,x) < \delta \Rightarrow \forall t \in \mathbb{N}\, d(F^t(y), F^t(x)) < \varepsilon$

REMARK

It is easy to see that equicontinuity is stronger than global stability. When $X = A^{\mathbb{Z}}$ the two definitions are equivalent and can be rewrite as follows

$\forall n \in \mathbb{N} \exists m \in \mathbb{N} \forall y, x \in A^{\mathbb{Z}}\, y_{[-m,m]} = x_{[-m,m]} \Rightarrow \forall t \in \mathbb{N}\, F^t(y)_{[-n,n]} = F^t(x)_{[-n,n]}$

DEF UNSTABLE SYSTEM

(X,F) is UNSTABLE if $\forall x \in X$, x is an UNSTABLE POINT

$\forall x \in X \exists \varepsilon > 0 \forall \delta \exists y \in X\, d(y,x) < \delta \wedge \exists t \in \mathbb{N}\, d(F^t(y), F^t(x)) \geq \varepsilon$

In $A^{\mathbb{Z}}$

$\forall x \in A^{\mathbb{Z}} \exists n > 0 \forall m \in \mathbb{N} \exists y \in A^{\mathbb{Z}}\, y_{[-m,m]} = x_{[-m,m]} \wedge \exists t \in \mathbb{N}\, F^t(y)_{[-n,n]} \neq F^t(x)_{[-n,n]}$

DEF SENSITIVITY TO THE INITIAL CONDITIONS

(X,F) is SENSITIVE TO THE INITIAL CONDITIONS if

$\exists \varepsilon > 0 \forall x \in X \forall \delta > 0 \exists y \in X\, d(y,x) < \delta \wedge \exists t \in \mathbb{N}\, d(F^t(y), F^t(x)) \geq \varepsilon$

In $A^{\mathbb{Z}}$

$\exists n \in \mathbb{N} \forall x \in A^{\mathbb{Z}} \forall m \in \mathbb{N} \exists y \in A^{\mathbb{Z}}\, y_{[-m,m]} = x_{[-m,m]} \wedge \exists t \in \mathbb{N}\, F^t(y)_{[-n,n]} \neq F^t(x)_{[-n,n]}$

REMARK

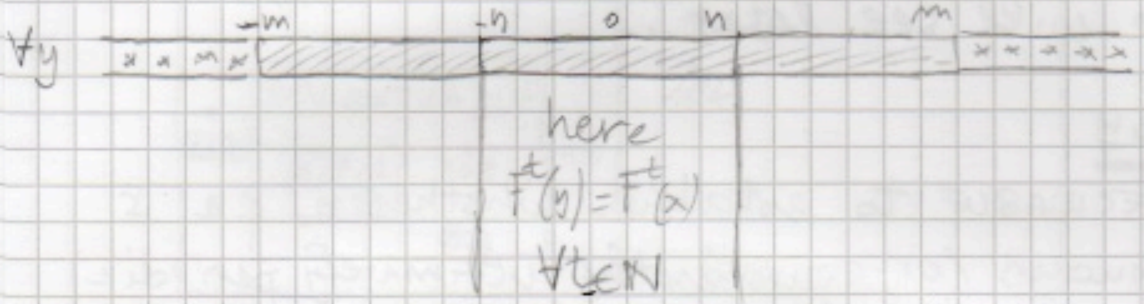
A sensitive system is an unstable system in which the constant $\varepsilon > 0$ does not depend on $x \in X$.

Such a constant is called SENSITIVITY CONSTANT

It is a specific feature of the system

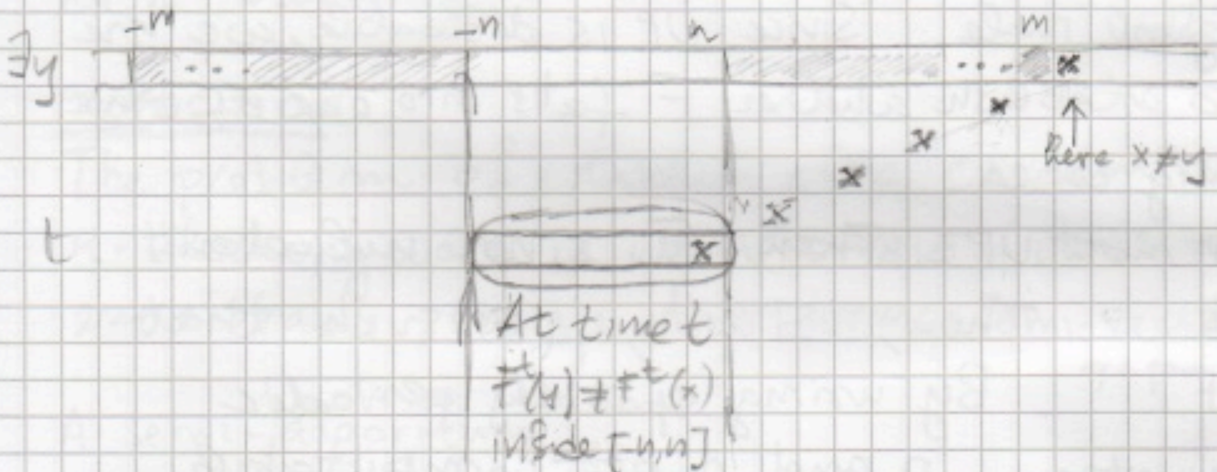
EQUICONTINUITY POINT x FOR A DTDS $(A^{\mathbb{Z}}, F)$

$$\forall n \in \mathbb{N}\ \exists m \in \mathbb{N}\ \forall y \in A^{\mathbb{Z}}\ y_{[-m,m]} = x_{[-m,m]} \Rightarrow \forall t\ F^t(y)_{[-n,n]} = F^t(x)_{[-n,n]}$$



UNSTABLE POINT x

$$\exists n \in \mathbb{N}\ \forall m \in \mathbb{N}\ \exists y \in A^{\mathbb{Z}}\ y_{[-m,m]} = x_{[-m,m]} \wedge \exists t\ F^t(y)_{[-n,n]} \neq F^t(x)_{[-n,n]}$$



SESITIVITY FOR A DTDS $(A^{\mathbb{Z}}, F)$

$$\exists n \in \mathbb{N}\ \forall x \in A^{\mathbb{Z}}\ \forall m \in \mathbb{N}\ \exists y \in A^{\mathbb{Z}}\ y_{[-m,m]} = x_{[-m,m]} \wedge \exists t\ F^t(y)_{[-n,n]} \neq F^t(x)_{[-n,n]}$$

A weaker notion of equicontinuity is the following

DEF ALMOST EQUICONTINUITY (A.E.)
(X, F) IS ALMOST EQUICONTINUOUS if the set $\mathcal{E}$ of its equicontinuity points is RESIDUAL ($\mathcal{E}$ is also dense)

MEANING OF RESIDUAL SET

EXAMPLE $\mathbb{I} = \bigcap_{\alpha \in \mathbb{Q}} (\mathbb{R} \setminus \{\alpha\})$ $\mathbb{I}$ is dense in $\mathbb{R}$

- $\mathbb{R} \setminus \{\alpha\} = (-\infty, \alpha) \cup (\alpha, +\infty)$ is open in $\mathbb{R}$
- $\mathbb{R} \setminus \{\alpha\}$ is dense in $\mathbb{R}$
- the intersection $\bigcap_{\alpha \in \mathbb{Q}} (\mathbb{R} \setminus \{\alpha\})$ is countable

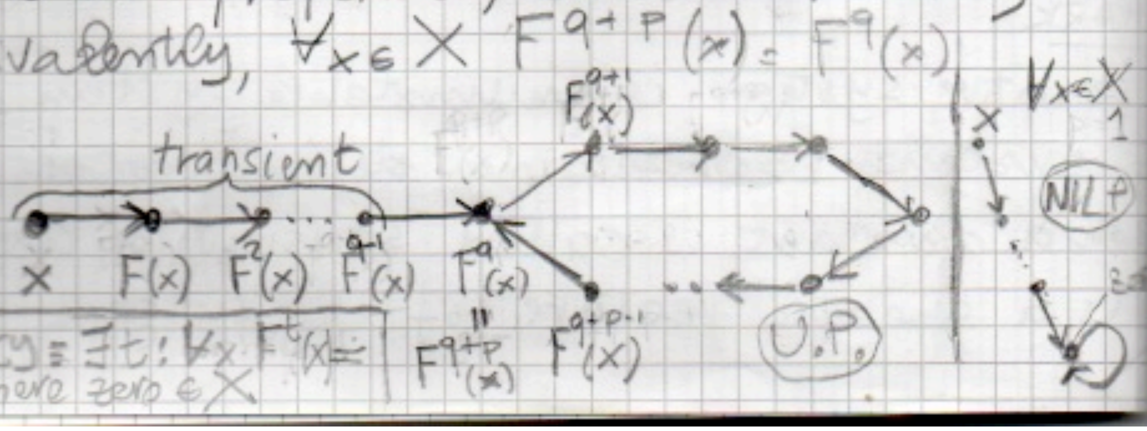
$\mathbb{I}$ also contains a countable intersection of open and dense subsets of $\mathbb{R}$

DEF Let (X, d) a metric space. Let $Y \subseteq X$. Y is residual if it contains a countable intersection of open and dense subsets of X

A stronger notion of equicontinuity is the following

DEF ULTIMATE PERIODICITY (U.P.) AND NILPOTENCY
(X, F) IS ULTIMATELY PERIODIC if there exist two naturals $p > 0$ (called the period) and $q \geq 0$ (called preperiod) such that $F^{q+p} = F^q$ or, equivalently, $\forall x \in X$ $F^{q+p}(x) = F^q(x)$

$\forall x \in X$



NILPOTENCY $\equiv \exists t : \forall x \; F^t(x) = $ zero where zero $\in X$

$\{x, F(x), \dots, F^{q-1}(x)\}$ is called transient and its length is just the preperiod q

$\{F^q(x), F^{q+1}(x), \dots, F^{q+p-1}(x)\}$ is called cycle and its length is just the period p

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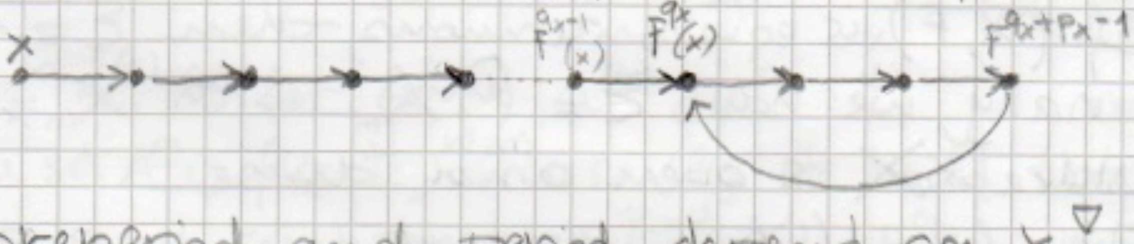
EXAMPLE

(X, F) with $|X| < \infty$

Necessarily, (X, F) is ultimately periodic.

Indeed, $\forall x \in X$ $\underbrace{\exists q_x \geq 0,\ p_x > 0 \text{ s.t. } F^{q_x+p_x}(x) = F^{q_x}(x)}_{x \text{ is ultimately periodic with prep. } q_x \text{ and period } p_x}$



the preperiod and period depend on x!

But, if one sets

$$q = \max \underbrace{\{q_x \mid x \in X\}}_{\text{finite set!}} \quad p = \operatorname{lcm} \underbrace{\{p_x \mid x \in X\}}_{\text{finite set!}}$$

We get that there exist a (common) preperiod q and a (common) period p such that $\forall x \in X$ $F^{q+p}(x) = F^q(x)$

Summarizing

x is ultimately periodic if $\exists q_x \geq 0, p_x > 0$ s.t. $F^{q_x}(x) = F^{q_x+p_x}(x)$

(X, F) is ultimately periodic if

$$\exists q \geq 0, p > 0 \text{ s.t. } \forall x \in X\ F^q(x) = F^{q+p}(x)$$

that is stronger than

$$\forall x \in X\ \underbrace{\exists q_x \geq 0\ p_x > 0 \text{ s.t. } F^{q_x}(x) = F^{q_x+p_x}(x)}_{x \text{ is ultimately periodic}}$$

PROPOSITION Let (X, F) be a DTDS

1) If (X,F) is U.P. $\overset{(*)}{\Longrightarrow}$ (X,F) is EQUICONTINUOUS

2) If (X,F) is EQUIC. $\Longrightarrow$ (X,F) is A.E.

(*) it is required that (X,d) is compact

proof

2) If (X,F) is equicontinuous, then $\mathcal{E} = X$.

Trivially, we have $\mathcal{E} = \bigcap_{m\in\mathbb{N}} X_m$ with $X_m = X$ where X is open and dense.

Therefore (X,F) is A.E.

1) We have to show that

$\forall \varepsilon>0\ \exists \delta>0\ \forall x,y\in X\ d(y,x)<\delta \Rightarrow \forall t\in\mathbb{N}\ d(F^t(y),F^t(x))<\varepsilon$

Choose arbitrarily $\varepsilon>0$.

Since $F^0 = Id, F^1, F^2, \ldots, F^{q-1}, F^q = F^{q+p}, \ldots, F^{q+p-1}$ are uniformely continuous functions

$[F^0]$ $\exists \delta_0>0\ \forall x,y\in X\ d(y,x)<\delta_0 \Rightarrow d(F^0(y),F^0(x))<\varepsilon$

$[F^1]$ $\exists \delta_1>0\ \forall x,y\in X\ d(y,x)<\delta_1 \Rightarrow d(F^1(y),F^1(x))<\varepsilon$

$\vdots$

$[F^q]$ $\exists \delta_q>0\ \forall x,y\in X\ d(y,x)<\delta_q \Rightarrow d(F^q(y),F^q(x))<\varepsilon$

$\vdots$

$[F^{q+p-1}]$ $\exists \delta_{q+p-1}>0\ \forall x,y\in X\ d(y,x)<\delta_{q+p-1} \Rightarrow d(F^{q+p-1}(y),F^{q+p-1}(x))<\varepsilon$

Then $\exists \delta = \min\{\delta_0, \delta_1, \ldots, \delta_q, \ldots \delta_{q+p-1}\}$ such that

$\forall x,y\in X,\ d(y,x)<\delta \Rightarrow \forall t\in\{0,\ldots,q+p-1\}\ d(F^t(y),F^t(x))<\varepsilon$

$\Rightarrow \forall t\in\mathbb{N}, d(F^t(y),F^t(x))<\varepsilon$

STABILITY / INSTABILITY IN CA WORLD

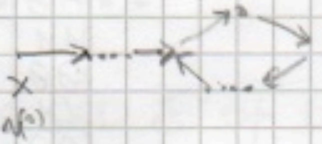
A preliminary remark

REMARK

Any spatially periodic configuration is ultimately periodic.
Indeed, let $x \in A^{\mathbb{Z}}$ be such that $\sigma^m(x) = x$ for some $m \in \mathbb{N}$.
Then, $x = {}^{\infty}u^{(0)\infty}$ for some $u^{(0)} \in A^m$. Let F be a CA.
Since the image of a spatially periodic configuration is again a spatially periodic configuration, we get

$F(x) = {}^{\infty}u^{(1)\infty}$ for some $u^{(1)} \in A^m$

$F^2(x) = {}^{\infty}u^{(2)\infty}$ for some $u^{(2)} \in A^m$

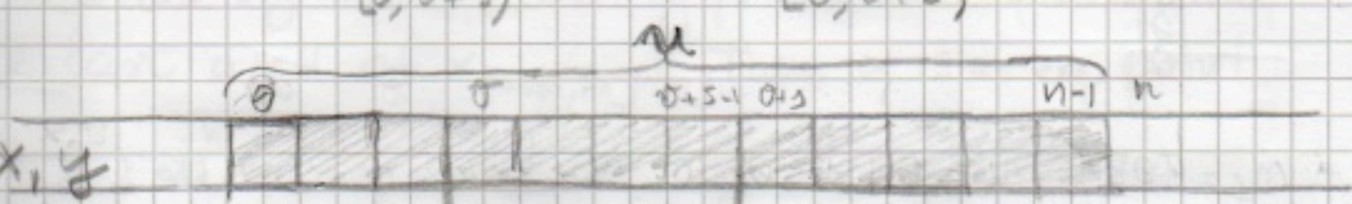


$F^t(x) = {}^{\infty}u^{(t)\infty}$ for some $u^{(t)} \in A^m$

Since the set A^m is finite, necessarily it hold that $\exists\, q_x \geq 0,\ p_x > 0$ such that $F^{q_x + p_x}(x) = F^{q_x}(x)$ or, equivalently, x is ultimately periodic.

DEF BLOCKING WORD FOR A CA F

Let $u \in A^m$ for some $m \in \mathbb{N}$. Let $s \in \mathbb{N}$ with $s > 0$.
The word u is s-BLOCKING if $\exists\, \sigma \in [0, m-s]$ (OFFSET) s.t. $\forall x, y \in A^{\mathbb{Z}}$: $x_{[0,m)} = y_{[0,m)} = u$ it hold that
$\forall t \in \mathbb{N}$ $F^t(x)_{[\sigma, \sigma+s)} = F^t(y)_{[\sigma, \sigma+s)}$



If $s \geq r$ u is a "wall": cells at the left side of u cannot interact with the ones at its right side

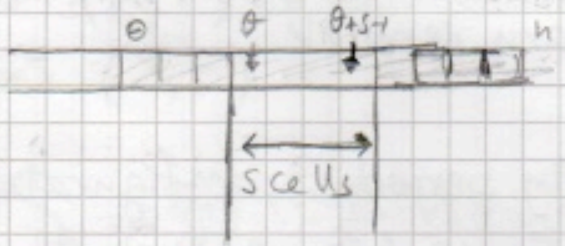
$\longleftrightarrow$ s cells
here $\forall t \in \mathbb{N}$ $F^t(x) = F^t(y)$

u produces a disconnection of the cell space $\mathbb{Z}$

REMARK $0 \to i$

EXAMPLES

	f_1	f_2	f_3	f_4
0 0 0	1	0	0	0
0 0 1	1	0	0	0
0 1 0	1	0	0	1
0 1 1	0	1	0	1
1 0 0	1	0	0	0
1 0 1	0	1	0	1
1 1 0	0	1	0	1
1 1 1	0	1	1	0
	↑ 23 maj	↑ 232 maj	↑ 128	↑ 108



• $m_f = 23$; $u = 00$ is 2-blocking with offset $\theta = 0$

* *	0 0	* *
	1 1	
	0 0	
	1 1	
	0 0	
	⋮	

$v = 11$ is also 2-blocking with $\theta = 0$

* *	1 1	* *
	0 0	
	1 1	
	0 0	
	⋮	

• $m_f = 232$; both $u = 00$ and $v = 11$ are 2-blocking with $\theta = 0$

* *	0 0	* *
	0 0	
	0 0	
	0 0	
	⋮	

* *	1 1	* *
	1 1	
	1 1	
	1 1	
	⋮	

• $m_f = 128$: $u = 0$ is 1-blocking with $\theta = 0$

* *	0	* *
	0	
	0	
	0	

• $m_f = 108$: $u = 01110$ is 3-blocking with $\theta = 1$

			u		
* *	0	0	0 1 1 1 0	0	* *
* *	0	0	1 0 1	0 0	* *
* *	0	0	1 1 1	0 0	* *
* *	0	0	1 1 1	0 0	* *
	0	0	1 0 1	0 0	* *

CASE 1

		u		
* *	0	0 1 1 1 0	1	*
	0	0 1 0 1 1	1	*
	0	0 1 1 1 0	*	*
		u		

if * = 1

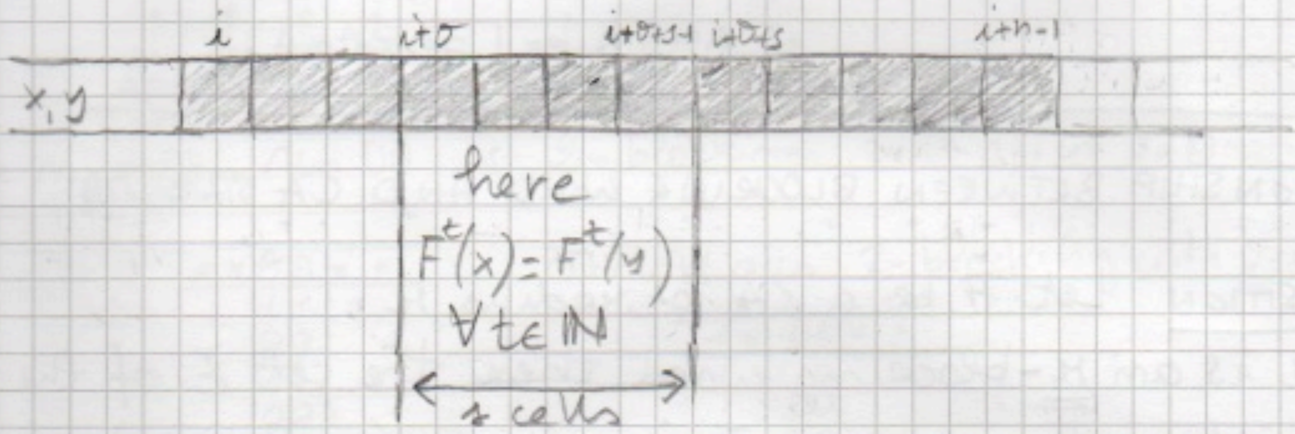
if * = 0

CASE 2

REMARK 1

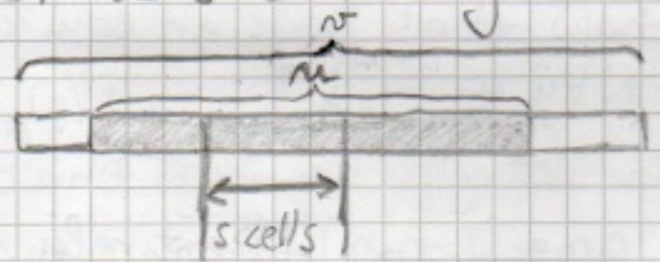
Since $F \circ \sigma = \sigma \circ F$, a word $u \in A^n$ is s-blocking if $\exists\, \vartheta \in [0, n-s]$ OFFSET such that $\forall i \in \mathbb{Z}$ $\forall x, y \in A^{\mathbb{Z}}$ $x_{[i,i+m-s)} = y_{[i,i+n-s)} = u$ it holds that

$\forall t \in \mathbb{N}$ $F^t(x)_{[i+\vartheta,\, i+\vartheta+s)} = F^t(y)_{[i+\vartheta,\, i+\vartheta+s)}$



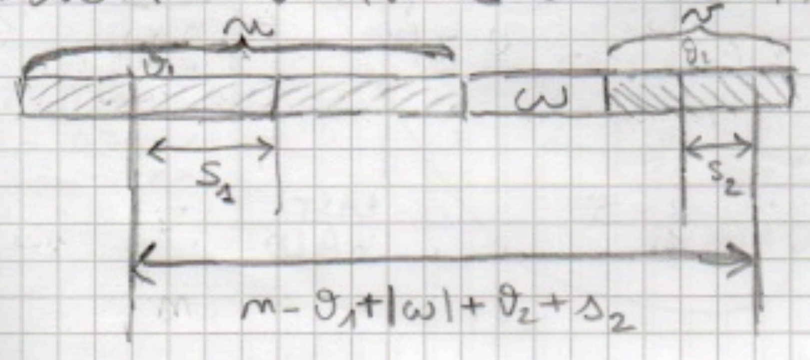
REMARK 2

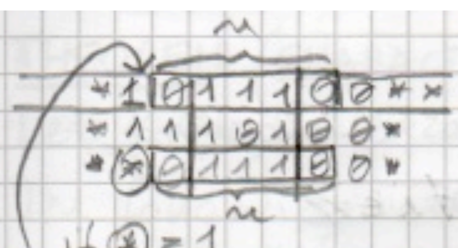
If $u \in A^n$ is an s-blocking word then every $v \in A^m$ with $m > n$ and containing u as factor is s-blocking too



REMARK 3

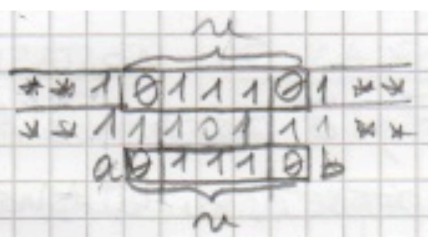
If $u \in A^n$ is an s_1-blocking word with offset ϑ_1 and $v \in A^m$ is an s_2-blocking word with offset ϑ_2 then $\forall w \in A^+$ uwv is a blocking word





if ⊛ = 1

if ⊛ = 0 ⟹ CASE 1



if $a=0 \wedge b=0 \Rightarrow$ CASE 1

if $a=0 \wedge b=1 \Rightarrow$ CASE 2

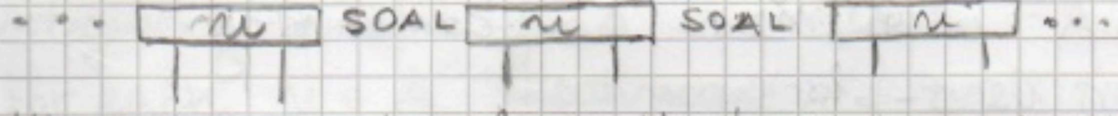
if $a=1 \wedge b=0 \Rightarrow$ CASE 3

if $a=1 \wedge b=1 \Rightarrow$ CASE 4

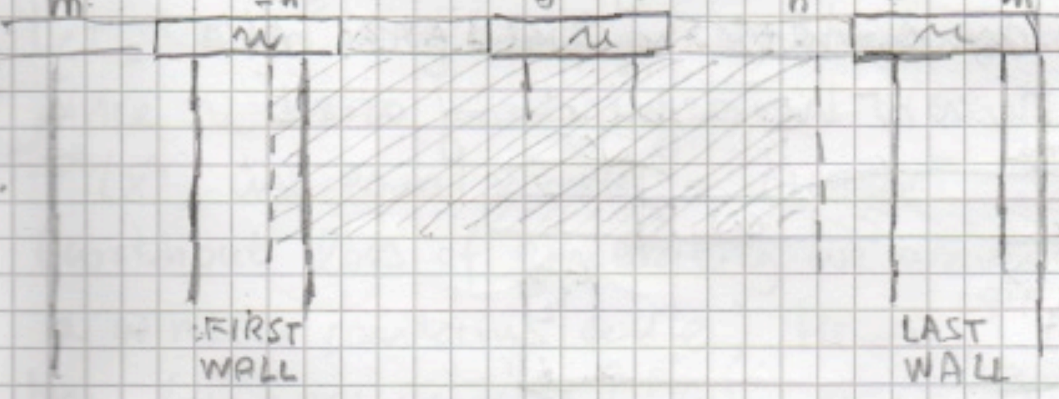
RELATIONSHIP BETWEEN BLOCKING WORD AND CA STABILITY

PROPOSITION Let F be a CA of radius r.
If u is an r-blocking word then the set $\mathcal{E}$ of the equicontinuity points of F is infinite (and dense)

proof (sketch) SOAL = "something of arbitrary length"
Let $x \in A^{\mathbb{Z}}$ be any configuration of the following form:



We are going to show that x is an equicontinuity point, i.e. $\forall n \in \mathbb{N}\ \exists m \in \mathbb{N}\ \forall y\ y_{[-m,m]} = x_{[-m,m]} \Rightarrow F^t(y)_{[-n,n]} = F^t(x)_{[-n,n]}$
Choose $n \in \mathbb{N}$ and let m be such that $[-m, m] \supseteq [-n,n]$ and $[-n,n]$ is entirely contained inside the segment starting from the first and the last wall of $x_{[-m,m]}$ - Then x is an eq. point.



THEOREM (CHARACTERIZATION OF CA EQUICONTINUITY)

Let $\langle A, r, f \rangle$ be a CA and let F be its global rule.

The following statements are equivalent:

(1) $(A^{\mathbb{Z}}, F)$ is equicontinuous

(2) $\exists k > 0$ s.t. $\forall u \in A^{2k+1}$ u is r-blocking

(3) $(A^{\mathbb{Z}}, F)$ is ultimately periodic

proof (3)=>(1) already proved for any DTDS (X, F)

(1)=>(2): We know that

$\forall m \in \mathbb{N}\ \exists k \in \mathbb{N}\ \forall x, y \in A^{\mathbb{Z}}\ x_{[-k,k]} = y_{[-k,k]} \Rightarrow \forall t \in \mathbb{N}\ F^t(x)_{[-m,m]} = F^t(y)_{[-m,m]}$

Let $n = r$. Then $\exists k \in \mathbb{N}$ s.t. $\forall x, y \in A^{\mathbb{Z}}\ x_{[-k,k]} = y_{[-k,k]} \Rightarrow$
$\forall t \in \mathbb{N}\ F^t(x)_{[-r,r]} = F^t(y)_{[-r,r]}$.

Thus any $u \in A^{2k+1}$ is $(2r+1)$-blocking and so it is also r-blocking.

(2)=>(3) We are going to show that there exist two integers $p > 0$ and $q \geq 0$ such that $F^{q+p} = F^q$

For each $u \in A^{2k+1}$, consider $x^u = {}^{\infty}u^{\infty}$ (which is a spatially periodic configuration). We know that $\{F^t(x^u)\}_{t \in \mathbb{N}}$ is ultimately periodic. Then $\{F^t(x^u)_0\}_t$ is ultimately periodic too. This also means that $\exists p_u > 0$ and $q_u \geq 0$ such that $F^{q_u + p_u}(x^u)_0 = F^{q_u}(x^u)_0$

Set $p = \text{l.c.m.}\{p_u \mid u \in A^{2k+1}\}$ and $q = \max\{q_u \mid u \in A^{2k+1}\}$

Therefore, $\forall u \in A^{2k+1}$ $F^{q+p}(x^u)_0 = F^q(x^u)_0$.

Let $x \in A^{\mathbb{Z}}$ any configuration and let $u = x_{[-k,k]}$

Since u is a blocking word $\forall t \in \mathbb{N}$ $F^t(x)$ and $F^t(x^u)$ are equal inside a window of length r (without loss of generality we assume that such a window contains cell 0). Hence $F^{q+p}(x)_0 = F^q(x)_0$

Now for every $i \in \mathbb{Z}$, $F^{q+p}(x)_i = \sigma^i(F^{q+p}(x))_0 = F^{q+p}(\sigma^i(x))_0$
$= F^q(\sigma^i(x))_0 = \sigma^i(F^q(x))_0 = F^q(x)_i$

EXAMPLES $m_f = 108$

	f
000	0
001	0
010	1
011	1
100	0
101	1
110	1
111	0

We already saw that 01110 is 3-blocking with offset 1 (and then it is 2-blocking)

Words in $B = \{00, 1111, 01110, 0110110, 010110, 011010, 10101\}$ are all 2-blocking:

```
*00*      *1111*     *0110110*     *010110*
 00        *00*       *11111*       *1111*
 :          00         *000*         *00*
            :           000           00
                         :             :

*011010*    *10101*    *01110*
 *1111*     *11111*      101
  *00*       *000*     *01110
   00        *000*
   :           :
```

Any word $u \in A^7$ contains at least one occurrence of some word from B.
Therefore, $(A^{\mathbb{Z}}, F)$ is equicontinuous with $q = 2$ and $p = 2$

$m_f = 0$. $\forall x \in A^{\mathbb{Z}}$ $F(x) = {}^{\infty}0^{\infty}$. F is nilpotent $\Rightarrow$ F is U.P. $\Rightarrow$ F is equicontinuous

$m_f = 204$. $\forall x \in A^{\mathbb{Z}}$ $F(x) = x$. F is U.P. with $q = 0 \wedge p = 1$ $\Rightarrow$ F is equicontinuous

THEOREM (CHARACTERIZATION OF CA ALMOST EQUIC.)

Let $\langle A, r, f \rangle$ be a CA and let F be its global rule.
The following statements are equivalent

(1) $(A^{\mathbb{Z}}, F)$ is not sensitive

(2) $(A^{\mathbb{Z}}, F)$ has an r-blocking word

(3) $(A^{\mathbb{Z}}, F)$ is almost equicontinuous

proof

(1) $\Rightarrow$ (2) Suppose that F is not sensitive, i.e.,
$\forall n \; \exists x \in A^{\mathbb{Z}} \; \exists m$ st $\forall y \; y_{[-m,m]} = x_{[-m,m]} \Rightarrow$
$\forall t \in \mathbb{N} \; F^t(y)_{[-n,n]} = F^t(x)_{[-n,n]}$
Choose arbitrarily n such that $2n+1 \geq r$
Then $\exists x \in A^{\mathbb{Z}}$ and $\exists m$ s.t. $\forall y \; y_{[-m,m]} = x_{[-m,m]}$
$\Rightarrow \forall t \in \mathbb{N} \; F^t(y)_{[-n,n]} = F^t(x)_{[-n,n]}$
Therefore, $u = x_{[-m,m]}$ is $(2n+1)$-blocking
(indeed $\forall z, z' \in A^{\mathbb{Z}}$ such that $z_{[-m,m]} = z'_{[-m,m]} = \overbrace{x_{[-m,m]}}^{u}$
it holds that $\forall t \; F^t(z)_{[-n,n]} = F^t(x)_{[-n,n]}$ and
$F^t(z')_{[-n,n]} = F^t(x)_{[-n,n]}$. Hence $\forall t \; F^t(z)_{[-n,n]} = F^t(z')_{[-n,n]}$)
Concluding, u is also r-blocking since $2n+1 \geq r$.

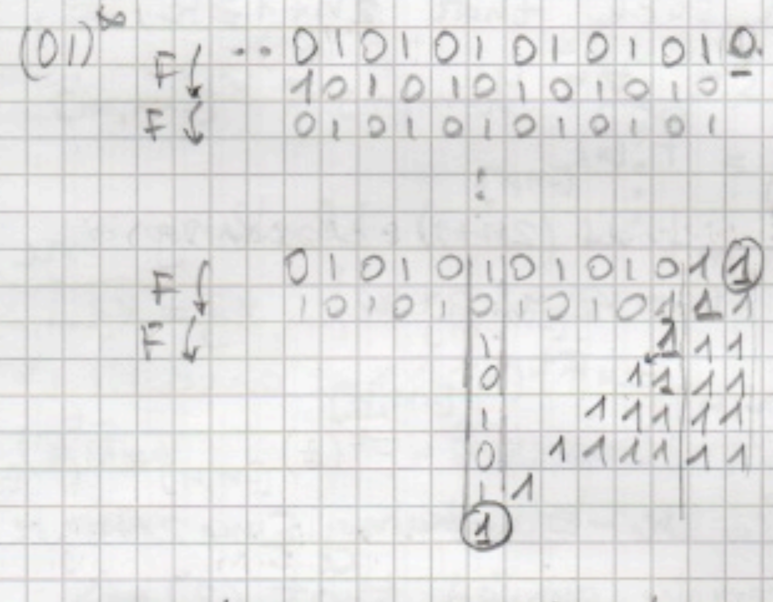
(2) $\Rightarrow$ (3) We have already proved that (2) $\Rightarrow$
there exist infinite equicontinuity points.
See Kůrka for the final part of the proof

(3) $\Rightarrow$ (1) Trivially if $(A^{\mathbb{Z}}, F)$ is almost equicont.
then it has one equicontinuity point and
so it cannot be sensitive.

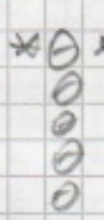
EXAMPLES $n_f = 232$ (majority)

$u = 00$ and $v = 11$ are 2-blocking $\Rightarrow$
$\exists$ an 1-blocking word $\Rightarrow$ F is almost eq.
$x = {}^{\infty}0^{\infty}$ is an equicontinuity point since
it contains infinite occurrences of u
$y = {}^{\infty}1^{\infty}$ is an equicontinuity point since
it contains infinite occurrences of v

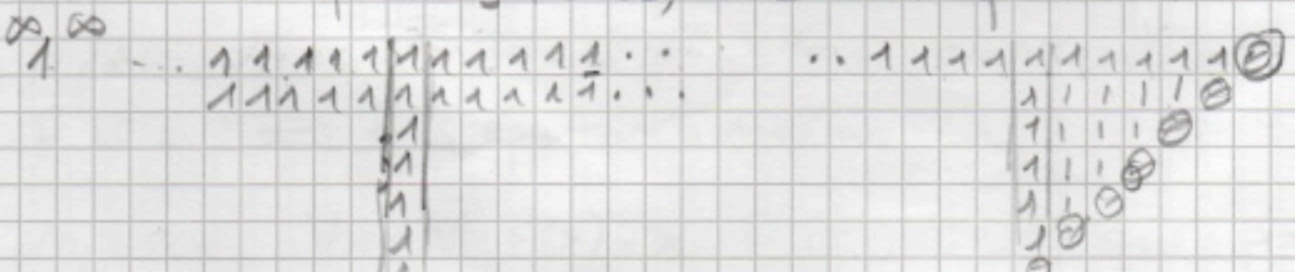
Is F equicontinuous? F is not because
there exists a configuration (${}^{\infty}(01)^{\infty}$) which
is not an equicontinuity point



$n_f = 128$ (product CA) $f(abc) = a \cdot b \cdot c$
$u = 0$ is a 1-blocking word



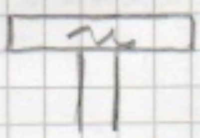
Then F is almost equicontinuous
but ${}^{\infty}1^{\infty}$ is not an equicontinuity
point. Hence, F is not equicontinuous



$n_f = 184$ (traffic CA)

$\forall u \in A^+$, u is not r-blocking ($r = 1$)

For a sake of argument, let us suppose that a certain $u \in A^+$ is 1-blocking



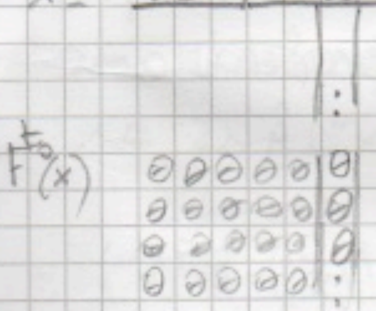
Consider $x, y \in A^{\mathbb{Z}}$ as follows

$x = \cdots 000\,|\;u\;|\,000 \cdots$ $\qquad$ $y = \cdots 111\,|\;u\;|\,000$

Clearly, by definition of f, the dynamical evolution $\{F^t(x)\}_{t \in \mathbb{N}}$ of x is such that $\exists t_0 : \forall t \geq t_0 \; F^t(x)_0 = 0$

$x = \; 000\,|\;u\;|\,000 \cdots$

all vehicles "1" inside u move to the right



Since x contains u and u is 1-blocking, it follows that $\forall t \geq t_0 \; F^t(y)_0 = 0$, but it is not possible. Indeed, necessarily "1"s at the left-side of y move into the wall under the action of F.

Therefore, there exist no 1-blocking word. Hence, F is sensitive to the initial conditions