

Introduction to Complex Systems

What is a Complex System?

Definition

A **complex system** is a multitude of simple components that, by cooperating, lead to a complex global behavior.

What is a Complex System?

Definition

A **complex system** is a multitude of simple components that, by cooperating, lead to a complex global behavior.

- Discrete models for complex systems

What is a Complex System?

Definition

A **complex system** is a multitude of simple components that, by cooperating, lead to a complex global behavior.

- Discrete models for complex systems
- Easy to implement

What is a Complex System?

Definition

A **complex system** is a multitude of simple components that, by cooperating, lead to a complex global behavior.

- Discrete models for complex systems
- Easy to implement
- Useful for studying emergence and dynamics

A paradigmatic model

- Cellular Automata

A paradigmatic model

- Cellular Automata

Goal

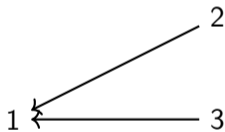
Understand how properties of a real phenomenon are reflected into mathematical properties of the model and study these latter.

Real phenomenon

- Reachability

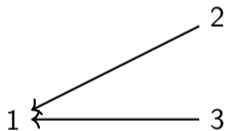
Real phenomenon

- Reachability



Real phenomenon

- Reachability

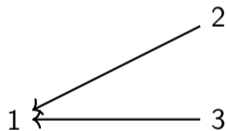


Model properties

- Surjectivity

Real phenomenon

- Reachability

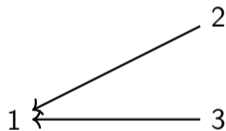


Model properties

- Surjectivity
- Transitivity

Real phenomenon

- Reachability



Model properties

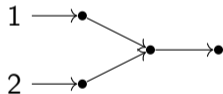
- Surjectivity
- Transitivity
- ...

Real phenomenon

- Non-collapsing orbits

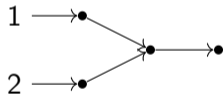
Real phenomenon

- Non-collapsing orbits



Real phenomenon

- Non-collapsing orbits



Model property

- Injectivity

Real phenomenon

- Reversibility

Real phenomenon

- Reversibility

Model properties

- Injectivity + Surjectivity

Real phenomenon

- Reversibility

Model properties

- Injectivity + Surjectivity

Real phenomenon

- Reversibility

Model properties

- Injectivity + Surjectivity

Reversibility

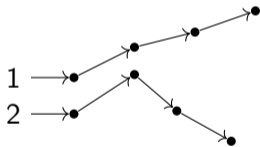
Reversibility corresponds to bijective dynamics described by the model itself.

Real phenomenon

- Instability

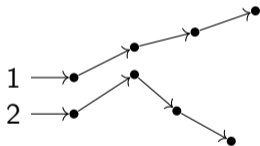
Real phenomenon

- Instability



Real phenomenon

- Instability

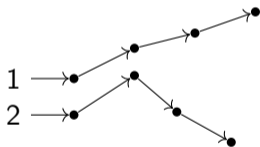


Model properties

- Lyapunov Instability

Real phenomenon

- Instability

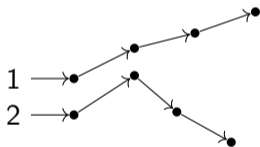


Model properties

- Lyapunov Instability
- Sensitivity to initial conditions

Real phenomenon

- Instability



Model properties

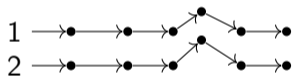
- Lyapunov Instability
- Sensitivity to initial conditions
- Chaos

Real phenomenon

- Stability

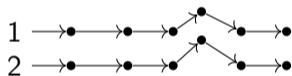
Real phenomenon

- Stability



Real phenomenon

- Stability

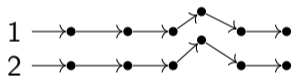


Model properties

- Lyapunov Stability

Real phenomenon

- Stability

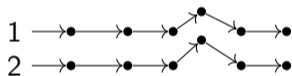


Model properties

- Lyapunov Stability
- Dense set of stability points

Real phenomenon

- Stability

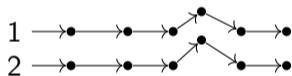


Model properties

- Lyapunov Stability
- Dense set of stability points
- Equicontinuity

Real phenomenon

- Stability



Model properties

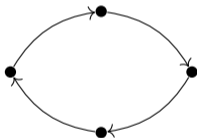
- Lyapunov Stability
- Dense set of stability points
- Equicontinuity
- ...

Real phenomenon

- Periodic behavior

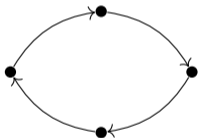
Real phenomenon

- Periodic behavior



Real phenomenon

- Periodic behavior

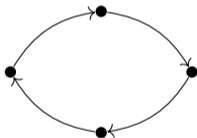


Model properties

- Ultimate periodicity

Real phenomenon

- Periodic behavior

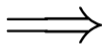


Model properties

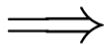
- Ultimate periodicity
- Dense set of periodic points

Real phenomenon (complex system)

Real phenomenon (complex system)

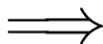


Real phenomenon (complex system)



Formal dynamical properties
of the model describing it

Real phenomenon (complex system)



Formal dynamical properties
of the model describing it

General Focus

Study complex collective behavior through a discrete mathematical model